COMPATIBLE TAPER AND VOLUME EQUATIONS FOR YOUNG LONGLEAF PINE PLANTATIONS IN SOUTHWEST GEORGIA

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Abstract—Inside and outside bark taper equations as well as compatible cubic foot volume equations were developed from felled tree data selected from young longleaf pine plantations that are part of an existing growth and yield study located in the Flint River drainage of southwest Georgia. A Max-Burkhart taper model was selected as the basic model form due to the accuracy found when applied to other southern pine species. Average diameter prediction error was approximately 0.25 inch and volume prediction error was less than 0.032 cubic feet.

INTRODUCTION

Taper and volume functions have been published for outside bark diameters (Baldwin and Polmer 1981) and inside bark diameters (Thomas and others 1995) for longleaf pine (*Pinus palustris* Mill.) plantations in central LA and east TX. Total and merchantable cubic foot volume equations have also been developed for plantations in this same region by Baldwin and Saucier (1983). Most published mensurational information on planted longleaf stands has been for cutover sites in the West Gulf physiographic region. Little is known regarding the growth and yield of longleaf plantations in southeast Georgia, especially for more intensively managed stands. Brooks and others (2002) presented an outside bark taper and a compatible cubic foot volume equation for young longleaf plantations in the southeast GA. The resultant taper equation was superior to the published equation by Baldwin and Polmer (1981). The resultant volume equation also showed more accurate volume estimates for this GA data set than equations published by Baldwin and Polmer (1981) or Baldwin and Saucier (1983).

The objective of this study was to develop inside and outside bark compatible taper and cubic foot volume equations as part of a growth and yield study for unthinned longleaf pine plantations on cutover sites in Southwest GA. Preliminary outside bark taper and volume equations based on the data used for the model developments discussed herein were published earlier (Brooks and others 2002).

DATA

Sample trees were selected during the summer of 2000 from three unthinned plantations in Dougherty and Worth Counties, GA that are part of an existing growth and yield study. Plantations were established in cutover stands that received mechanical as well as chemical site preparation. Plantations ranged in age from 12 to 14 years and were established on sandy loam soils using bare root seedlings. Approximately 15 sample trees were selected from the interior of each plantation from the area buffering existing permanent growth and yield plots. Trees possessing multiple stems, broken tops, obvious cankers or crooked boles were not included in the sample. Each sample tree selected for stem analysis was felled at ground level. A 100-foot tape was used to directly measure total height of each tree following felling (recorded to the nearest 0.1 foot). Diameter outside bark (d.o.b.) at breast height was measured and recorded to the nearest 0.1 inch. Each tree was bucked into sections and one inch thick sample disks were obtained at different heights above the tree base. Disks were extracted at the base, 0.5, 2.0, 4.5, 6.0 feet and at 4 foot intervals to a two-inch top diameter outside bark. Each disk was labeled and sealed in a plastic bag to preserve moisture and prevent shrinkage. In the laboratory, diameter outside and inside bark of each disk were measured. The data set included 420 inside and outside bark measurements on 42 sample trees. Actual volume for each bolt and tree was calculated using the overlapping bolts method as described by Bailey (1995) and a generalized Newton formula described by Wiant and others (1992). Sample tree distribution by height and diameter class is displayed in table 1.

TAPER AND VOLUME EQUATION

The Max and Burkhart (1976) segmented polynomial taper equation was selected for use in this study. This equation is well-known and widely used in the United States. This equation is of the form:

$$\frac{d^2}{D^2} = b_1(Z-1) + b_2(Z^2-1) + b_3(a_1-Z)^2 I_1 + b_4(a_2-Z)^2 I_2 \quad (1)$$

Table 1—Distribution of sample trees by d.b.h. and total height class

d.b.h. class (in)	Total height class (ft) 20	25	30	35	40	Totals
2	5	3				8
3		4	1			5
4		2	5	5		12
5			1	4	2	7
6				5	1	6
7				1	3	4
Totals	5	9	7	15	6	42

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where:

$$I_i = \begin{cases} 1 & Z \le a_i \\ 0 & Z > a_i \end{cases} \qquad i = 1, 2$$

$Z = h/H$

h = height above the ground to the measurement point (feet),

H = total tree height (feet),

D = diameter outside bark at breast height (inches),

d = diameter outside bark to measurement point at height h (inches),

a_i = join points to be estimated from the sample data.

$i = 1, 2$,

b_1, b_2, b_3, b_4 : regression coefficients.

A volume equation was derived by integrating the Max and Burkhart taper equation:

$$kD^2H \begin{Bmatrix} \frac{b_2}{3}(Z_u^3 - Z_l^3) + \frac{b_1}{2}(Z_u^2 - Z_l^2) - (b_1 + b_2)(Z_u - Z_l) \\ -\frac{b_3}{3}[(a_1 - Z_u)^3 J_1 - (a_1 - Z_l)^3 K_1] \\ -\frac{b_4}{3}[(a_2 - Z_u)^3 J_2 - (a_2 - Z_l)^3 K_2] \end{Bmatrix} \qquad (2)$$

where:

k = 0.0054542,

$Z_l = h_l/H$,

$Z_u = h_u/H$,

h_l = lower height of interest (feet),

h_u = upper height of interest (feet),

$$J_i = \begin{cases} 1 & Z_u \le a_i \\ 0 & Z_u > a_i \end{cases} \qquad i = 1, 2$$

$$K_i = \begin{cases} 1 & Z_l \le a_i \\ 0 & Z_l > a_i \end{cases} \qquad i = 1, 2$$

All other variables as previously defined.

MODEL EVALUATION

To evaluate model performance, average bias, the standard error of the estimate and a fit index, as described by Schlaegel (1981), were employed for model evaluation. These evaluation statistics are defined as:

$$Average\ Bias = \frac{\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)}{n}$$

$$SEE = \sqrt{\frac{\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2}{\text{n-k}}}$$

$$FI = 1 - \left[\frac{\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2}{\sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)^2}\right]$$

where:

Y_i = observed value for the i^th observation,

$\hat{Y}_i$ = predicted value for the i^th observation,

$\bar{Y}$ = mean of the Y_i,

k = the number of estimated parameters,

n = number of observations in the dataset,

SEE = the standard error of the estimate,

F = fit index.

To concurrently minimize error in compatible taper and volume, the taper and volume equations were fitted simultaneously using SUR procedure in SAS (SAS Institute 2002). All parameters were shared by both the taper and volume equations. The models were independently fitted to both outside and inside bark data.

RESULTS

Taper Equation

Parameter estimates were obtained by simultaneously fitting the taper and volume equations (outside bark and inside bark). All parameters in the equation were found to be significant at the 0.0001 level (tables 2 and 3). The overall statistics of fit (average bias, SEE and FI) for the entire merchantable stem were calculated and presented in table 4 for both diameters outside and inside bark. The results indicate that equation 1 explained more than 97 percent of the variation in diameter outside and inside bark. Equation 1 was further evaluated by relative height (h/H) classes in order to evaluate its performance at different positions throughout the merchantable stem. Since stem analysis was stopped at a 2-inch top diameter outside bark, the data were split into nine relative height classes. Average bias and SEE were calculated for each equation by relative height class and used to evaluate taper and volume (outside bark and inside bark) estimates (table 5). The results indicate that equation 1 performed well for all sections. Average error (SEE) ranged from 0.2 to 0.3 inch for diameter outside and inside bark.

Volume Prediction

Statistics of fit (average bias, SEE and FI) for the entire merchantable stem volume (outside and inside bark) are presented in table 4. Equation 2 explained more than 96 percent of the variation for predicting volume outside and inside bark. The results indicate that equation 2 had better overall prediction statistics for volume with lower average biases, SEE and higher FI. Volume prediction by relative height class was also evaluated (table 5). Equation 2 performed well for all sections. Average error (SEE) ranged from 0.016 to 0.042 cubic feet for volume outside bark and between 0.013 and 0.034 cubic feet for volume inside bark.

Table 2—Parameter estimates, standard errors and *P*-values for planted longleaf pine outside bark taper and volume equations

Parameter	Estimate	Standard Error	*P*-value
b_1	-2.7307	0.3314	<0.0001
b_2	1.1566	0.1935	<0.0001
b_3	-0.7712	0.1828	<0.0001
b_4	380.9826	61.5103	<0.0001
a_1	0.6457	0.0823	<0.0001
a_2	0.0339	0.0026	<0.0001

Table 3—Parameter estimates, standard errors and *P*-values for planted longleaf pine inside bark taper and volume equations

Parameter	Estimate	Standard Error	*P*-value
b_1	-2.4164	0.2037	<0.0001
b_2	1.0841	0.1204	<0.0001
b_3	-1.2913	0.1187	<0.0001
b_4	235.6075	56.9886	<0.0001
a_1	0.6047	0.0396	<0.0001
a_2	0.0373	0.0043	<0.0001

Table 4—Fit statistics of each equation in the taper and volume equations

Equation	Avg. Bias	SEE	FI
Dob	0.0455	0.2633	0.9723
V_{ob}	0.0041	0.0311	0.9664
Dib	-0.0010	0.2301	0.9726
V_{ib}	0.0020	0.0244	0.9651

V_{ob} *= volume outside bark*
V_{ib} *= volume inside bark*

Table 5—Bias and standard error of the estimate by relative height (RH) class for the inside and outside bark taper and volume equations

		Dob (in)		V_{ob} (ft³)		Dib (in)		V_{ib} (ft³)	
RH	n	Bias	SEE	Bias	SEE	Bias	SEE	Bias	SEE
0.0-0.1	121	0.069	0.290	0.004	0.016	-0.009	0.248	0.003	0.013
0.1-0.2	65	-0.054	0.173	-0.005	0.033	-0.048	0.167	-0.001	0.025
0.2-0.3	39	0.053	0.205	0.009	0.040	-0.012	0.188	0.003	0.033
0.3-0.4	30	0.035	0.234	0.007	0.032	0.025	0.224	0.003	0.024
0.4-0.5	51	0.070	0.313	0.006	0.042	0.031	0.268	0.002	0.034
0.5-0.6	37	0.083	0.308	0.006	0.041	0.023	0.251	0.002	0.032
0.6-0.7	33	0.040	0.323	0.002	0.041	0.008	0.299	0.000	0.033
0.7-0.8	31	0.056	0.298	0.008	0.026	0.010	0.253	0.004	0.019
0.8-0.9	13	0.111	0.248	0.013	0.020	0.036	0.202	0.008	0.014

CONCLUSIONS

In this study, a system of taper and volume equations were developed for young longleaf pine in southwest GA. To ensure numeric consistency, a simultaneous fitting procedure was used. Parameter estimates were obtained that simultaneously minimized taper and volume error. Equations 1 and 2 showed consistent performance in terms of overall fit statistics, sectional performance, average bias and SEE in estimating diameter and volume, respectively. In the future, weight equations (green wood and bark, green wood, dry wood) will be developed using density-integral approach and compatible taper, volume and weight equations system will be derived.

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