

DEVELOPMENT OF A SHORLEAF PINE INDIVIDUAL-TREE GROWTH EQUATION USING NON-LINEAR MIXED MODELING TECHNIQUES

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Abstract—Nonlinear mixed-modeling methods were used to estimate parameters in an individual-tree basal area growth model for shortleaf pine (*Pinus echinata* Mill.). Shortleaf pine individual-tree growth data were available from over 200 permanently established 0.2-acre fixed-radius plots located in naturally-occurring even-aged shortleaf pine forests on the USDA Forest Service Ozark and Ouachita National Forests in western Arkansas and southeastern Oklahoma. The plots were established during the period from 1985 to 1987. Two subsequent re-measurements were used in this study, resulting in a total of three measurements and two growth periods. The equation can be used to predict the annual basal area growth of individual shortleaf pine trees as a function of individual tree basal area, forest stand basal area per acre, stand age, and the ratio of individual tree d.b.h. to quadratic mean stand d.b.h. The mixed-model procedure used random plot-level effects associated with individual tree basal area to account for correlation among individual trees located on the same plot. This resulted in improved fit statistics compared to a similar model fitted using nonlinear ordinary least squares.

INTRODUCTION

Shortleaf pine (*Pinus echinata* Mill.) has the widest geographical distribution of any southern pine, yet has received less attention in terms of growth modeling than other southern pines. Most growth and yield information for southern pine prior to 1985 was based on full stocking assumptions. Lynch and others (1999) used re-measured plot data from over 200 permanently established shortleaf pine growth study plots to develop a distance-independent individual-tree simulator for naturally-occurring shortleaf pine forests. This simulator utilized an individual tree basal area growth equation in which parameters were estimated by ordinary least squares (OLS). The OLS procedure performed satisfactorily in tests but does not fully account for correlations among individual sample trees located on the same sample plot. Trincado and Burkhart (2006) suggested that the assumption of correlated errors could be relaxed in the presence of a tree-level random effect in a stem profile curved fitted to data consisting of multiple stem measurements on a sample of individual trees. Here we wish to use a plot-level random effect to help account for correlations among individual trees located on the same plot. The development of mixed-modeling estimation techniques (e.g., Gregoire and others 1995, Hall and Bailey 2001) and the availability of additional plot measurements not used in the 1999 analysis provide the opportunity to obtain new estimates for the parameters of a basal area growth equation using mixed modeling methods.

METHODS

Lynch and others (1999) estimated the following nonlinear model parameters using OLS:

$$G_i = \frac{b_1 B_i^{b_2} - (b_1 B_i / B_M^{1-b_2})}{1 + \exp(b_3 + b_4 B_s + b_5 A + b_6 R_i + b_7 B_i)} \quad (1)$$

where G_i is annual basal area growth (square feet) of tree i ; B_i is basal area (square feet) of tree i ; A is stand age; R_i is the ratio of quadratic mean stand diameter to the d.b.h. of tree i ; B_s is stand basal area (square feet per acre); $B_M = 7.068384$ square feet (the maximum expected basal area for a shortleaf pine in managed stands); and b_1, b_2, \dots, b_7 are parameter estimates. Testing of the tree-level independent variables in the equation above indicated that the most promising independent variable with which to associate a plot-level parameter was individual tree basal area. This modification resulted in a mixed-effects model having a random parameter associated with individual tree basal area. The “mixed model” is a result of fixed parameters, which are constant for all plots, and random parameters, which vary by (randomly chosen) plots.

RESULTS AND CONCLUSIONS

Nonlinear mixed modeling techniques were used to estimate parameters in the individual-tree basal area growth model for shortleaf pine. The NLME procedure with SPLUS software described by Pinheiro and Bates (2000) was used to obtain the parameter estimates. Fit statistics indicated improved fit for the mixed-effects model compared to a more traditional approach that did not include mixed effects. Thus, mixed-model estimation procedures appear to be advantageous for individual tree basal area growth equations where data are obtained by re-measured plot sampling.

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