A MIXED-EFFECTS HEIGHT-DIAMETER MODEL FOR COTTONWOOD IN THE MISSISSIPPI DELTA

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ABSTRACT

Eastern cottonwood (*Populus deltoides* Bartr. ex Marsh.) has been artificially regenerated throughout the Mississippi Delta region because of its fast growth and is being considered for biofuel production.This paper presents a mixed-effects height-diameter model for cottonwood in the Mississippi Delta region. After obtaining height-diameter measurements from the plot/stand of interest, a mixed-effects model can be calibrated often improving height estimates relative to an uncalibrated fixed-effects model. When using an independent validation dataset, the calibrated mixed-effects height-diameter model vastly improved height predictions compared to a completely fixed-effects model. When using only one tree in calibration, bias decreased from -1.1164 m to -0.1334 m while the mean square error (MSE) decreased from 2.3421 to 0.4869 for the fixed-effects and mixed-effects models, respectively. When using three trees in calibration, the bias and MSE were reduced to -0.0495 m and 0.3012. The use of three trees in model calibration will likely provide a reasonable compromise between predictive ability and field sampling times.

INTRODUCTION

Height-diameter models are an integral component of forest inventories and in many cases reduce sampling times. Using diameter (D) to predict height (H) has long been implemented since D is more efficiently measured than H yet the two are strongly correlated. Mixed models are becoming a popular modeling tool to provide more site specific predictions. Many regionwide mixed model equations have been fit that can then be calibrated for local site conditions. When compared to the traditional means of developing local H-D equations, where H and D are measured and then a separate equation is fit for a tree, plot, stand, or tract, a mixed-effects model analysis is efficient because a model can be calibrated without having to statistically fit a model and thus even small sample sizes can be used (since degrees of freedom are not a concern).

Both linear and nonlinear mixed models have been used to model the H-D relationship for many species including loblolly pine (*Pinus taeda* L) in the southeastern US (Trincado and others 2007, VanderSchaaf 2008), cherrybark oak (*Quercus pagoda* Raf.) in the Western Gulf (Lynch and others 2005), and stone pine (*Pinus pinea* L.) in Spain (Calama and Montero 2004). Mixed-effects models provide an efficient means to obtain cluster-specific, or for this particular example, plot-specific, parameters through the prediction of cluster-specific random effects. For example, total tree height (H) can be predicted as a function of diameter at breast height (D):

$$\ln H_{ki} = \beta_0 + \beta_1 \ln D_{ki} + \varepsilon_{ki} \qquad [1]$$

Where:
ln--natural logarithm,
H_{ki}--total tree height (m) of tree i for plot k,
D_{ki}--diameter at breast height (cm) of tree i for plot k,
β_0, β_1--parameters to be estimated,
ε_{ki}--random error where it is assumed $\varepsilon \sim N(0, \sigma^2 I)$.

Equation [1] provides what is often termed a population-average estimate of H for a given D. The parameters β_0 and β_1 are assumed to be fixed, or that the parameter estimates apply to every experimental unit (e.g. every tree) in a population. Whether trees are located in North Carolina or Arkansas, the parameter estimates are assumed to be correct. However, plot-specific characteristics such as soil type, nutrient status, elevation, aspect, competition from herbaceous vegetation, genetic stock, etc., may result in the parameters differing across plots. Thus, specific plots may have what are generally termed "random parameters" in mixed-effects model terminology. Equation [1] can be altered by adding plot-specific random effects to the population-average parameters to produce plot-specific parameters:

$$\ln H_{ki} = (\beta_0 + u_{0k}) + (\beta_1 + u_{1k}) \ln D_{ki} + \varepsilon_{ki} \qquad [2]$$

Where:
u_{0k}, u_{1k}--are plot-specific random effects, assumed to be $N(0, \sigma^2_0)$ and $N(0, \sigma^2_1)$, respectively,
$(\beta_0 + u_{0k})$--plot-specific intercept,
$(\beta_1 + u_{1k})$-- plot-specific slope,
and all other variables as previously defined.

Additionally, a covariance, σ_{01}, can be assumed to exist between u_{0k} and u_{1k}. Linear mixed-effects models, in this particular case, produce an efficient estimate of plot-specific parameters because only six parameters are estimated using the model fitting algorithm ($\beta_0, \beta_1, \sigma^2_0, \sigma^2_1, \sigma_{01}, \sigma^2$). Based on the variance and covariance estimates, plot-

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specific random effects (u_{0k}, u_{1k}) can be predicted and then added to the population-average intercept and slope (β_0, β_1) estimates to obtain plot-specific parameter estimates. Plot-specific random parameter estimates produce a more localized H-D equation since the random effects account for local site conditions such as soil type, genetic stock, site preparation, mid-rotation silvicultural practices, spatial and time-specific climatic conditions, etc. The prediction of plot-specific random effects is conducted outside the model fitting algorithm and thus degrees of freedom are not lost. A less efficient means of obtaining plot-specific parameter estimates would be to estimate parameters separately for each plot.

Although the parameter estimation efficiency of mixed models is an advantage, often the greatest advantage is the ability to calibrate the model using data independent of those used in model fitting. Hence, for trees obtained from plots not in the model fitting dataset, plot-specific (or stand-specific) H-D relationships can be produced if H and D observations have been collected from trees in that plot (or stand).

Eastern cottonwood (*Populus deltoides* Bartr. ex Marsh.) is a fast-growing tree species (Cooper 1990) and has recently been one of the most widely artificially regenerated tree species in the Mississippi Delta region. It has several commercial uses including pulpwood and a strong consideration as a species of choice for biofuel production. The objectives of this paper are to present a mixed-effects individual tree H-D model for cottonwood established in the Mississippi Delta region and to demonstrate how this model improves height predictions for an independent cottonwood validation dataset.

METHODS

DATA USED IN MODEL FITTING

Observed H-D pairs were obtained from a study site occupying about 3 ha located on the University of Arkansas Pine Tree Branch Experiment Station in St. Francis County, AR (Stuhlinger and others 2010). The soil is a Calloway silt loam and the site was previously used as row cropland. See Table 1 for summary information of the data used in model fitting.

STUDY DESIGN AND LAYOUT

The study design was a replicated randomized complete block consisting of six blocks that contained nine plots of randomly assigned cottonwood clones. Each plot contained 56 trees in a seven x eight tree layout. The interior 30 trees (five x six tree layout) were measured, leaving an unmeasured buffer around each measurement plot.

STUDY ESTABLISHMENT

The site was sprayed prior to planting using applications of Goal® (oxyfluorfen) and Roundup® (glyphosate) to control weeds. Each planting row was sprayed with a liquid fertilizer prior to planting at a rate of 112 kg per ha of nitrogen. The site was then bedded (51-cm high beds) to facilitate furrow irrigation.

Cottonwood cuttings were planted by hand at a 3.1 x 3.1 m spacing in March 1996. Disking, herbicide spraying, and hand weeding continued for two years after the initial planting. Irrigation with well water was conducted each year whenever a 5-cm rainfall deficiency was reached. Deficiencies occurred on average five times per year, resulting in about 8 to 10 ha-cms of irrigation water per year.

Nine cottonwood clones were tested, two from Texas (S7C15 and S13C20), five from Stoneville, Mississippi (ST72, ST124, ST148, ST163, and Delta View (mix of the four ST clones)), and two hybrids of eastern cottonwood and black cottonwood (Populus trichocarpa Torr. and Gray) from the northwestern U.S. (49-177 and 1529).

Poor survival forced the complete replanting of five cottonwood clones (S7C15, ST72, ST148, ST163, and Delta View) in Spring 1997. This resulted in two separate groups of clones: replanted clones, which grew 9 years, and non-replanted clones, which grew for 10 years. Measurement data for the two groups were combined when estimating parameters of equations [1] and [2]. Total tree height and DBH (ages 3, 5, 10 for the non-replant cohort and ages 4 and 9 for the replant cohort) were measured for all surviving trees.

MODEL DEVELOPMENT AND PARAMETER ESTIMATION

Prior to model fitting, all trees with broken and leaning stems were removed from the analysis. Observations were also checked to ensure that the H-D relationships were biologically reasonable (Figure 1) and that errors in data recording and translation to a computer file were not made (e.g. the elimination of an H of 4 m and a D of 45 cm).

Parameters of equations [1] and [2] were estimated using SAS Proc MIXED (Littell and others 1996) which assumes random errors are normally distributed and subsequently estimates parameters using maximum likelihood. Rather than simply assuming β_0 and β_1 were random across plots, likelihood ratio tests were conducted and Akaike Information Criterion (AIC) values were examined to determine if assuming β_0 was random, β_1 was random, and if assuming a covariance term existed between u_{0k} and u_{1k} (σ_{01}), produced better model fit statistics. For this analysis, since the reduced models are nested within the full model, a Likelihood ratio test is appropriate (Schabenberger and

Pierce 2002; pgs. 547, 557). Under the null hypothesis, the test statistic is assumed to follow a χ^2_{df} distribution where df is the difference between the number of fixed-effects parameters in the full and reduced models and hence the critical value also differs at an level of 0.05.

In many cases random effects account for nearly all autocorrelation among observations when using longitudinal datasets (Trincado and Burkhart 2006, VanderSchaaf and Burkhart 2007); however, a modeler can also directly model the random error structure. Within-cluster temporal correlation was ignored because it is assumed these models will be calibrated using temporary plot data or at a particular point in time for plots that are repeatedly measured. Hence, for this analysis, each measurement age of a plot is considered a separate plot. When estimating parameters in a mixed-effects model framework it is important to consider that the measurement intervals may not be the same among the model fitting and prediction/validation datasets - this can cause problems because a covariance structure that is appropriate for the model fitting dataset may not be appropriate for the model prediction/calibration dataset. Additionally, no attempt was made to model spatial correlation among trees to reduce complexity when users apply this model. For this particular study, the random error covariance-variance matrix was assumed to be $\sigma^2 I_{nk}$.

DATA USED IN MODEL VALIDATION

To quantify if model calibration of equation [2] produces superior height estimates relative to equation [1], an independent validation dataset was used (Table 1, Figure 1). Heights and Ds were obtained from a cottonwood study adjacent to the study used in model fitting. However, unlike the model fitting dataset, the validation data study site was not irrigated. A previous report demonstrated that irrigation resulted in substantially different growth patterns (Stuhlinger and others 2010), thus observations from the unirrigated study can be considered an independent dataset from that used in model fitting. Besides the lack of irrigation, the only difference in terms of study design, layout, and establishment between the two datasets was that the unirrigated study plots were subsoiled prior to planting and the liquid nitrogen fertilizer was injected 51 cm below the soil surface.

MODEL VALIDATION

Validation of equation [2], firstly, consisted of randomly sampling various numbers of trees from each plot to predict plot-specific random effects for the validation dataset. Secondly, the predicted plot-specific random effects were then added to the population-average parameters (as estimated using the model fitting dataset) to produce predicted plot-specific random parameters of the validation dataset. After obtaining predicted plot-specific parameters, equation [2] and equation [1] (an entirely fixed-effects model) were used to predict height for all trees not used in calibration of equation [2]. To provide a more conservative comparison between equation [1] and equation [2] for various model calibration sample sizes, for those trees used in calibrating equation [2], it is assumed that those heights are also known when calculating model validation statistics for equation [1].

Validation analyses follow those presented in Trincado and others (2007). The difference between the observed (Hobs) and predicted height (Hpred) of all trees whose heights were predicted for each individual plot (k), age (j), and replication (r – as explained below, for each plot, age, and sample size combination 10 random selections were conducted) was calculated for both equations ($e_{kjri} = Hobs_{kjri} - Hpred_{kjri}$). For each plot (k), age (j), and replication (r) combination, the mean residual (emean) and the sample variance (v) of residuals were computed and considered to be estimates of bias and precision; respectively. An estimate of mean square error (MSE) was obtained for each equation by combining the bias and precision measures using the following formula:

$$MSE_{kjr} = [emean_{kjr}]^2 + v_{kjr} \qquad [3]$$

Values of MSE were compared between equations [1] and [2] to determine which model produced better estimates of height for this particular cottonwood validation dataset. It is well known that logarithmic transformations in many cases help to linearize data and produce homogeneity of variances; however, a transformation bias occurs since additive errors in log-log models become multiplicative when transformed back to the original scale. To account for the transformation bias, the procedure recommended by Baskerville (1972) was used:

$$\ln H_{ki} = \beta_{0k} + \beta_{1k} \ln D_{ki} + \sigma^2/2 \qquad [4]$$

Where:
σ^2 -- mean square error (or residual variance) from the model fit (0.02365 for equation [1] and 0.00429 for equation [2], see Table 2).

For equation [1], $\beta_{0k} = \beta_0$ and $\beta_{1k} = \beta_1$ for all k; respectively. All validation statistics presented in this paper are based on untransformed errors.

The numbers of trees randomly selected from a particular plot for a certain age to be used in calibrating equation [2] were 1, 2, 3, 5, and 10. These sample sizes represent practical numbers of trees to be measured while conducting field inventories. To ensure that bias, variance, and MSE measures between the varying number of randomly selected trees are coherent (for example, if a plot only contains 4 trees, it cannot be used in model calibration for sample sizes of 5 and 10), only those plots that contained at least 20 trees at a specific age were selected. Hence, measuring 10 trees per plot would involve measuring at a minimum half of all trees in a plot. For those plot ages selected, the number of trees per plot varied from 20 to 29.

It should be noted that, in practice, predicted random effects for a particular plot and age are statistics themselves (and thus each predicted plot-specific random effect has a sampling distribution for a particular sample size) and can vary depending on what trees from a particular plot were used in model calibration. Similar to Trincado and others (2007), to capture variability among potential random effects predictions for a particular plot, age, and sample size, for each model calibration sample size trees were randomly selected 10 times. When calculating model validation statistics, all 10 samples for each model calibration sample size from each of the 39 plot observations were averaged (resulting in one MSE, emean, and v observation for each plot and age combination – or 39 observations). The average of these 39 observations was then calculated for each sample size to compare among model equations. To ensure that the specific trees selected for a particular model calibration sample size were coherent, the tree used to calibrate equation [2] for a particular plot and age when using a sample size of one was also used to calibrate the model for a particular plot and age when using a sample size of two, and so forth.

For the case where all tree heights were predicted using equation [1], there was no need to conduct 10 separate replications. The emean, v, and MSE were calculated for each of the 39 plots and then these observations were averaged.

RESULTS AND DISCUSSION

Based on the model fitting results (Table 2), it is best to assume that both β_0 and β_1 are random (or, essentially, that each plot (or stand) has their own intercept and slope) and that a covariance (σ_{01}) exists between u_{0k} and u_{1k}.

VALIDATION RESULTS

As the number of trees used in model calibration increased (Mixed-Effects model – equation [2]) the three validation statistics decreased in magnitude (Table 3). For the fixed-effects model (equation [1]), the most conservative comparison between the mixed- and fixed-effects results are when any tree used in model calibration is also assumed to have been measured when simply using the fixed-effects model. Else, how do you know whether the calibration statistics are better due to model calibration or because some of the trees in the plot had heights actually measured. For all n_c, the mixed-effects model MSE was at least 78 percent less (ranging from 78 percent to 87 percent) than the corresponding fixed-effects model MSE.

In some cases, individuals may predict heights for all trees within a plot (or use equation [1] to predict heights for every tree in a plot). For this case, it is correct to compare the fixed-effects model MSE when using a $n_c = 0$ (MSE = 2.3421) to all mixed-effects model MSEs. For all n_c, the mixed-effects model MSE was at least 79 percent less (ranging from 79 percent to 92 percent) than the fixed-effects model MSE.

In terms of choosing an optimal model calibration sample size, a reasonable trade-off between statistical measures (precision and accuracy) and sampling times appears to be three trees. Calama and Montero (2004) recommended using four trees for stone pine. Sample sizes of 5 and 10 trees do improve statistical measures but will require substantially more sampling time, especially measuring 10 trees. Even the use of only 1 tree in calibration substantially improves height estimates. Similar results were observed by Calama and Montero (2004) and Trincado and others (2007).

CONCLUSIONS

A mixed-effects H-D model for cottonwood in the Mississippi Delta region was presented. By obtaining H-D measurements from plots/stands of interest, Equation [2] can be calibrated to local site conditions. When using an independent validation dataset, the calibrated H-D model (equation [2]) was shown to vastly improve height predictions compared to an entirely fixed-effects H-D model (equation [1]). Large increases in the predictive ability of equation [2] were observed when using only 1 tree in calibration; however, the use of 3 trees in model calibration will likely provide a reasonable compromise between predictive ability and field sampling times.

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Table 1—Tree-level summary statistics of eastern cottonwood plantings at the five ages used in model fitting and the four ages used in model validation. Std. dev. is the standard deviation. For the model fitting dataset, data were obtained from a total of 54 plots (six blocks x nine clones). For the model validation dataset, data were obtained from 39 plot observations (plots had to have at least 20 trees for a particular sampling age – if a plot was included at age 3, it was not necessarily included at age 5). n is the number of trees

Model Fitting Dataset

		D (cm)				H (m)			
Age	*n*	Min	Mean	Max	Std. dev.	Min	Mean	Max	Std. dev.
3	496	2.2	9.1	13.4	1.82	4.1	7.9	15.0	1.34
4	504	1.3	9.5	15.7	2.77	2.0	8.0	12.0	1.79
5	420	2.8	14	19.1	2.14	4.6	11.1	14.0	1.18
9	492	2.5	17.1	28.4	4.1	4.7	15.8	21.1	2.68
10	371	9.9	19	30.5	3.57	8.8	15.9	21.5	2.57

Model Validation Dataset

		D (cm)				H (m)			
Age	*n*	Min	Mean	Max	Std. dev.	Min	Mean	Max	Std. dev.
3	304	2.2	7.5	11.3	1.62	3.4	6.2	9.9	1.03
4	246	2.3	7.4	12.2	1.97	3.5	5.8	7.9	0.98
5	160	4.6	11.9	15.2	1.78	6.2	8.9	11.1	1.12
9	182	4.6	12.1	17.7	2.1	5.5	10.1	12.8	1.27

Table 2—Population-average (β_0 and β_1) and random effects variance (σ^2_0, σ^2_1) and covariance (σ_{01}) parameter estimates. Where: -2LL -- twice the negative log-likelihood (smaller is better), AIC -- Akaike's Information Criterion (smaller is better), σ^2 -- estimated mean square error. Critical values for Full versus Reduced model analyses at an alpha level = 0.05 are: 3 df -- 7.81, 2 df -- 5.99, 1 df -- 3.84, where df is the number of fixed effects in the Full model minus the number of fixed effects in the Reduced model. For instance, when comparing the FULL model to the Fixed-Effects model the df = 6 – 3 = 3, since estimates of σ^2_0, σ^2_1 , and σ_{01} are required for the FULL model. There were a total of 2283 observations used in model fitting and the total number of clusters (plots) was 133

	Fixed-Effects		Random β_0		Random β_1		Random β_0, β_1		FULL	
	Est	SE	Est	SE	Est	SE	Est	SE	Est	SE
β_0	0.3311	0.0199	1.0381	0.0238	0.9723	0.0163	1.1566	0.0276	1.2410	0.0448
β_1	0.8123	0.0078	0.5298	0.0069	0.5483	0.0090	0.4820	0.0101	0.4524	0.0159
σ^2_0	-	-	0.0337	-	-	-	0.0450	-	0.1808	-
σ^2_1	-	-	-	-	0.0050	-	0.0051	-	0.0215	-
σ_{01}	-	-	-	-	-	-	-	-	-0.0541	-
-2LL	-2054.0		-4914.1		-4782.1		-5004.2		-5111.1	
AIC	-2052.0		-4910.1		-4778.1		-4998.2		-5103.1	
σ^2	0.02365		0.00516		0.00551		0.00461		0.00429	

Table 3—Model validation summary statistics when using varying numbers of trees randomly selected from the model validation dataset to calibrate equation [2]. The Fixed-Effects model is equation [1]. A total of 39 plot and age combinations were used. To eliminate the dependence of the model validation statistics on one random sample, for each calibration sample size (n_c) and plot and age combination, trees were randomly selected 10 times

	Fixed-Effects model			Mixed-Effects model		
n_c	Bias (m)	Variance	MSE	Bias (m)	Variance	MSE
0	-1.1164	0.6558	2.3421	-	-	-
1	-1.0693	0.6961	2.2488	-0.1334	0.2565	0.4869
2	-1.0245	0.7290	2.1531	-0.0689	0.2408	0.3602
3	-0.9787	0.7599	2.0591	-0.0495	0.2301	0.3012
5	-0.8862	0.8010	1.8688	-0.0324	0.2084	0.2483
10	-0.6588	0.8069	1.3994	-0.0193	0.1620	0.1804

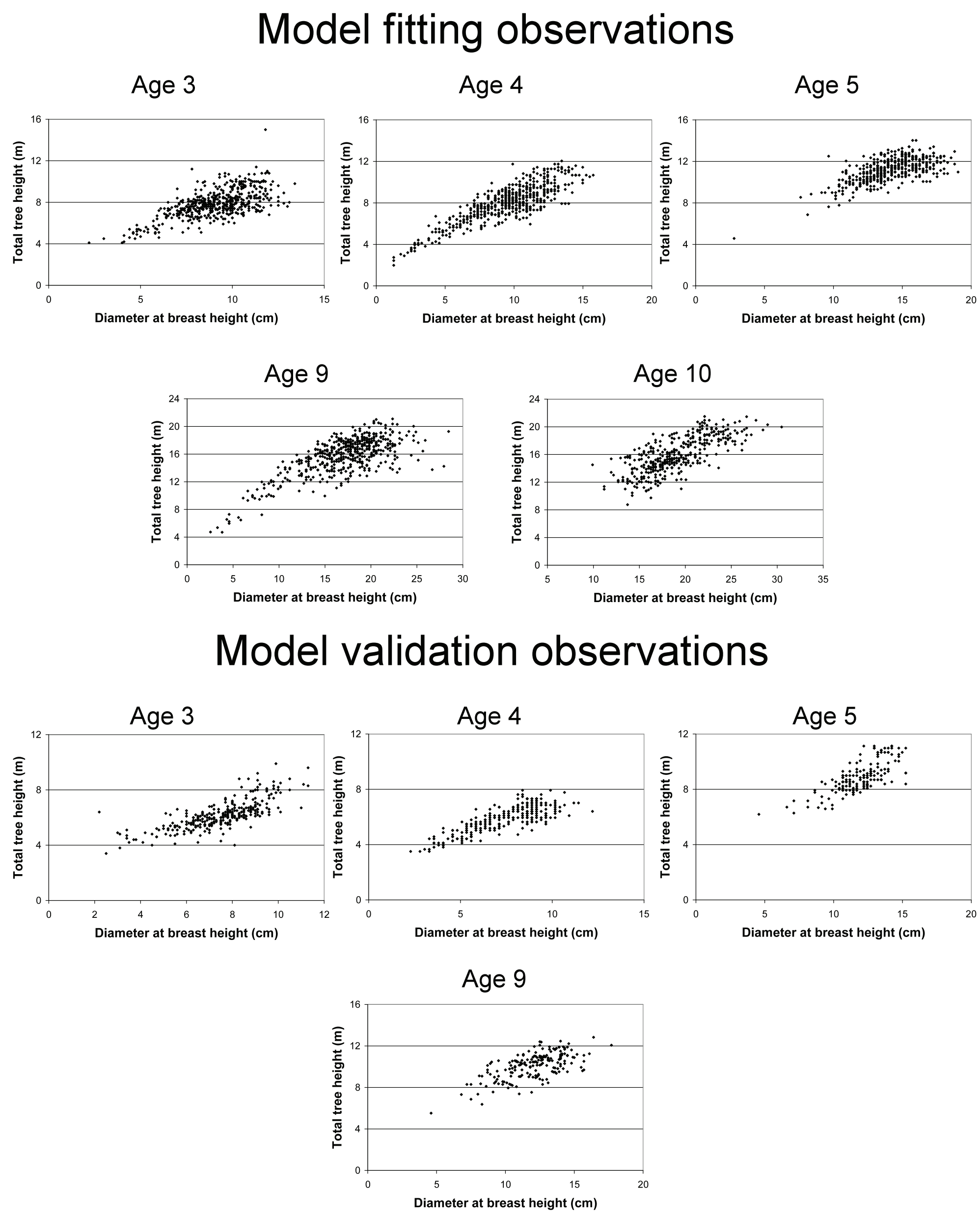

Figure 1—The top figure depicts the height-diameter relationship by measurement age for the model fitting dataset (n = 496 for age 3, n = 504 for age 4, n = 420 for age 5, n = 492 for age 9, n = 371 for age 10). The bottom figure depicts the height-diameter relationship by measurement age for the model validation dataset (n = 304 for age 3, n = 246 for age 4, n = 160 for age 5, n = 182 for age 9).