Modelling Regeneration Success and Early Growth of Forest Stands

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THE IMPORTANCE OF INITIAL HEIGHT GROWTH IN QUAKING ASPEN HEIGHT GROWTH MODELS EXPRESSED AS 2nd ORDER DIFFERENTIAL EQUATIONS

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ABSTRACT

In a study of height growth patterns of quaking aspen (Populus tremuloidies Michx.) in Minnesota, we represented the height-age pattern using a second order ordinary differential equation with environmentally governed parameters. Solving a second order differential equation that has been converted to a system of two first order equations requires knowledge of, or information on, initial conditions for both state variables, height and height growth. We used the natural boundary condition when age is zero, height is zero, i.e., \( h(t=0) = 0 \). Initial conditions for the second state variable, height growth, were estimated when fitting the equations to observations. This paper reports our research to predict the initial height growth from stem analysis data, and to assess the sensitivity of predicted height to differences in initial condition estimates. We discovered that initial height growth is the single most important 'parameter' to be estimated in our model.

INTRODUCTION

Height is an important property of forest trees and stands from several points of view: species succession (taller trees may spread the most propagules and have an advantage), forest mensuration (percent of height as a spacing indicator, Eichhorn's rule), and site assessment, where theory and practice have claimed that tree height is an important signal for suitability of the growing environment (height-age, height-diameter relations can be used as indicators of site quality).

Representing the height-age relation is more of a modeling challenge than one might expect because when empirical data are looked at closely, the pattern is more complex than is described by Spurr (1952). We conjectured that tree height growth could be represented by identifying a simple basic model that makes use of more advanced mathematical methods (an integro-differential equation), with focus shifting from getting flexibility out of the model to identifying the environmental factors governing model parameters.
The parameters are the superposition constants that result from linearizing the model in equation [3] using a Taylor series expansion in function space (Bellman and Kalaba 1965). The predicted heights in equation [4] are computed by numerically solving one case of the inhomogeneous equation and as many cases of the homogeneous form as there are unknown initial conditions (because there are four unknown initial conditions \([a, b, c, g_0]\)) of the homogeneous equation of the linearized form of equation [3]. Parameters \(a, b, c\) and their variances can be derived from the variance-covariance matrix for the \(a\)'s, using standard linear least squares regression methods. Significance tests for parameters \(a, b, c\) also follow standard methods for linear regression (Compte 1994). Although it may appear very complex, the procedure for estimating unknown initial conditions using measurements on one state variable as boundary conditions can be formulated and solved using a common spreadsheet program. Of course, the linearization must be done by hand for spreadsheet solution.

Three categories of estimation problems arose: a) what are the environmental variables governing the parameters, b) what are the algebraic forms of these relationships, and c) what governs the initial height growth (which must either be measured or estimated)?

The parameter estimation problem was formed as a multi-point boundary value problem in ordinary differential equations, wherein model parameters are converted to state variables with zero right hand sides (equation [1] above) (Bellman and Kalaba 1965, Leary 1970). Because the parameters are converted to state variables, and there are several products of state variables, the problem becomes a nonlinear multi-point boundary value problem. The unknowns are the initial conditions for the trivial differential equations in equation [3], and the initial height growth, \(g_0\). The boundary conditions are the heights of different trees on different plots at different ages.

The boundary value problem approach to parameter estimation allows larger estimation problems to be handled than, say, the simplex method. Our largest problem called for simultaneous estimation of 226 unknown parameters in 150 simultaneous first order equations. Because of the large computational load we converted from solving differential equations to solving first order forward difference equations.

Our scientific hypotheses, focused on parameters in equation [3], were:
- \(a\) is governed by moisture availability
- \(b\) is governed by nutrient availability
- \(c\) is governed by heat.

**MATERIALS**

To test the above hypotheses we assembled existing stem analysis data for quaking aspen from a study by Schlaegel (Schlaegel 1975), and a more recent study reported on by Alban et al. (1991). Field and office procedures are reported in these publications. An additional 13 plots were located across northern Minnesota to span a range of climate and soil fertility conditions (Miller and Cooper 1993). On these additional plots four trees growing within 50'
Figure 1. a) Residual pattern (observed - predicted) when equation [3] was fit to pooled data for all trees. Number of parameters estimated was 4. b) Residual pattern when equation [3] was fit to data for each tree individually. Number of parameters estimated per tree was 4, for a total of 300.

We conjectured that we could approach the narrow residual pattern of Figure 1a) by including information on the annual climate and soil conditions at the growing site and associating these conditions with parameters a,b,c. We feel we were successful in determining that parameter c is governed by heat at the growing site expressed as growing degree days above 41°F, and that parameter a is governed by estimated actual annual evapotranspiration. We also determined that while parameter b has a statistically significant relation with the nutrient synecological coordinate, it is not practically significant, once growing degree days and estimated actual evapotranspiration are associated with c and a, respectively. Our nutrient data were probably inadequate given the very high spatial variability of soil chemical properties in our region (Alban 1974, Mroz and Reed 1991).

To our surprise, we found that in predicting future heights the initial height growth, \( \hat{g}_0 \), was the most important single 'parameter' in the model. Our discovery was gratifying on one hand because initial height growth can be measured with simple equipment. However, it also caused concern because it is known that initial tree height growth can be extremely variable (Palik and Pregitzer 1995).

We turned our attention to finding simple ways to estimate the important parameter, \( \hat{g}_0 \). Of course, we were not present to actually measure the early height growth of each tree in our data set, so we don't have actual measured tree heights to use as a dependent variable. Our 'observations' of early heights were estimates based on using Carmean's method of interpolating the location of the terminal bud between discs cut at either two or four foot intervals above a one half foot stump. The interpolation process made these 'observations'
fourth, and first and fifth disks above breast height. The six mean height growth estimates were calculated for each individual tree, converted to plot average values and graphed against the mean of model-based estimated initial height growth, $\hat{g}_0$ for each plot. Ground line as the reference height gave a smaller variance than when breast height was used as the reference height. For our observations, the relation between estimated height growth based on ages and heights at the 2nd and 4th disks cut above ground line gave estimates of $g_0$ with the lowest variance (Figure 2). There were on average, about 5 years difference between the age at the second and fourth disks, and about 3 meters difference in height. Application of the just described procedure presumes one can, in sucker stands over 5 years of age, visually approximate five years worth of past height growth. Mean annual height growth is used in the following equation to predict the estimated initial height growth, $\hat{g}_0$:

$$\hat{g}_0 = 1.394 \left( 1 - e^{-0.903 \text{ (mean annual height growth between disks 2 and 4) }} \right),$$

uncorrected fit index = .973, corrected fit index = 0.712 (Wilkinson 1990).

![Figure 2. Relation between early growth rate between the 2nd and 4th disks cut from trees, and estimates of initial height growth, $g_0$, equation [3]. All heights are in meters.](image)

The sample point to the right in Figure 2 is for trees in our sample from Pike Bay (Minnesota). Their very rapid growth lead us to suspect that they might have been of stump sprout origin, rather than root sucker. If the single point corresponding to Pike Bay West is removed from the dataset, the relation between annual height growth between the 2nd and 4th disks, and $\hat{g}_0$, is linear (equation [6]):
DISCUSSION

On the basis of our study results to date we conclude that having a good estimate of initial quaking aspen height growth is more important for estimating future height than having a complete history of either heat sums or actual evapotranspiration spatially interpolated to the plot location.

In application, getting good estimates of initial height growth may be difficult, especially if quaking aspen are over about 6 years of age. (One can estimate previous year’s height growth from morphological features of the stem for a few years down from the leading shoot, but probably not over about 5 years.) Splitting the log longitudinally is one alternative, but we could have used this approach only on trees from our study. (The Schlaegel and Alban et al. study trees had been left in the woods to rot about 20 years previously. Further, it is not clear how well splitting the lower bole of quaking aspen would work as the leading shoot and terminal bud are not as prominent as for most conifers.) Periodic, temporary inventories of regenerating aspen stands would seem to be the most reliable source of the needed information.

The methodology of differential and difference equations is applicable from any set of initial conditions. For example, the methodology developed could be applied to an aspen stand currently 10 years old. We would need to abandon the natural boundary condition, $h(t=0)=0$, and develop estimates for both $h(t=10)$ and $g(t=10)$. The former might be measurable with a height pole. The latter may be estimated from height measurements between, say, 4 or 5