Equations Relating Compacted and Uncompacted Live Crown Ratio for Common Tree Species in the South

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ABSTRACT

Species-specific equations to predict uncompacted crown ratio (UNCR) from compacted live crown ratio (CCR), tree length, and stem diameter were developed for 24 species and 12 genera in the southern United States. Using data from the US Forest Service Forest Inventory and Analysis program, nonlinear regression was used to model UNCR with a logistic function. Model performance was evaluated with standard fit statistics (root mean squared error, mean absolute error, mean error, and model efficiency) and by comparing the results of using the observed and predicted UNCR values in secondary applications. Root mean squared error for the regression models ranged from 0.062 to 0.176 UNCR and averaged 0.114 UNCR across all models. Height to live crown base calculations and crown width estimations based on the observed and predicted UNCR values were in close agreement. Overall, the models performed well for the *Pinus* and *Taxodium* genera and several individual hardwood species; however, model performance was generally poor for the *Acer*, *Quercus*, and *Carya* genera.

Keywords: Forest Inventory and Analysis, crown base height, crown modeling, nonlinear regression, forest inventory

Beyond stem diameter and tree height, live crown ratio (LCR) is one of the most commonly used tree metrics in forestry applications. LCR, the proportion of total tree length supporting live foliage, has a myriad of uses at both the individual tree and stand level. At the individual tree level, LCR is used to predict crown widths (Bechtold 2003, 2004) and diameter and height growth (Forest Vegetation Simulator Staff (FVS) 2001). At the stand level, LCR is used in silviculture prescriptions (O’Hara and Oliver 1999), fire behavior models (Scott and Reinhardt 2001), bird and wildlife habitat assessments (MacArthur and MacArthur 1961, Hunter 1990), scenic beauty and stand structure visualizations (Ribe 1989, McGaughey 2004), and growth predictions (Sprinz and Burdick 1987). LCR ranges from 0 to 1 and is often expressed as a percentage. Trees with LCR near 0 have very little foliage, whereas trees with LCR near 1 have foliage along most of the bole.

The Forest Inventory and Analysis (FIA) program of the US Forest Service includes two measures of LCR in its national forest inventory: compacted crown ratio (CCR) and uncompacted crown ratio (UNCR) (Schomaker et al. 2007, US Forest Service 2007). When CCR is measured, observers visually rearrange the foliage of the tree so that wide spaces between branches in the upper part of the crown are filled. With UNCR, there is no visual rearrangement of the branches. For some applications (e.g., growth predictions), either UNCR or CCR can be used; however, for other applications (e.g., fire behavior) UNCR may be more appropriate.

FIA protocols call for CCR to be measured on all forested ground plots, whereas UNCR is a required measurement on only a subset of the same plots; that is, each individual FIA region has the option of measuring UNCR on all plots (Schomaker et al. 2007, US Forest Service 2007). To fill in the data gap where UNCR was not measured, Monleon et al. (2004) and Toney and Reeves (2009) developed equations to predict UNCR for tree species in the western United States. In a similar manner, the objective of this study was to develop equations to predict UNCR on the basis of CCR and other easily measured tree or stand attributes for trees in the southern United States.

**Methods**

**Data**

The national network of systematically located FIA ground plots is divided into two phases: phase 2 and phase 3. Phase 2 plots are located across the country at a spatial intensity of approximately one plot per 6,000 ac (Bechtold and Patterson 2005). The phase 3 plots are a 1/16 subset of the phase 2 plots; therefore, each phase 3 plot represents approximately 96,000 ac (Bechtold and Patterson 2005). Many tree attributes are measured at each plot. Those of interest for this study include stem diameter, tree length, CCR, and UNCR. Diameter, length, and CCR are measured on all phase 2 and phase 3 plots. UNCR is measured on all phase 3 plots and is optionally measured on phase 2 plots. At present, UNCR is not measured on phase 2 plots in the South.

For most trees, stem diameter was measured at breast height (4.5 ft); however, diameters were measured at the groundline or root collar, whichever was higher, for a small number of "woodland" species with a shrublike form. For all trees, tree length was measured from ground level to the top of the tree. If a tree had a missing top, the length from the ground to the existing tree top was recorded as the actual length, and the length from the ground to the estimated location where the missing top would have been was recorded as the total length (US Forest Service 2007). To obtain UNCR, the length...
from the top of the live foliage to the base of the obvious live crown was divided by actual tree length, where the base of the obvious live crown was defined as “the point on the tree where most live branches above that point are continuous and typical for a tree species (and/or trees size) on a particular site” (US Forest Service 2005, p. 3). Branches below the obvious live crown but within 5 ft of it were included in the live crown length if they were 1.0 in. or larger in diameter at the base above the swelling where they joined the main stem or larger branch (US Forest Service 2005). CCR was measured as the length of the compacted live crown divided by the actual tree length. The length of the compacted live crown was determined by a visual transfer of the “lower live branches to fill in large holes in the upper portion of the tree until a full, even crown is visualized” (US Forest Service 2007, p. 92). Field crews were instructed not to “over-compact trees beyond their typical full crown situation” (US Forest Service 2007, p. 92). Both CCR and UNCR were measured to the nearest 1% for all live trees 5 in. or more in diameter. UNCR and CCR were converted from a percentage to a proportion for this study.

For this study, data from all FIA phase 3 plots measured in 11 southern states (Alabama, Arkansas, Florida, Georgia, Kentucky, Louisiana, North Carolina, South Carolina, Tennessee, Texas, and Virginia) between 2003 and 2008 were obtained. Following Monleon et al. (2004) and Toney and Reeves (2009), trees with CCR greater than 0.90 were excluded from the analysis because UNCR cannot be smaller than CCR and must therefore be between CCR and 1.0. Also omitted from the analyses were trees with missing tops (i.e., actual length ≠ total length) and trees with “no crown” by the FIA definition, such as might occur after severe damage (US Forest Service 2005). These eliminated trees amounted to approximately 1.5% of the initial data set. The analysis was further limited initially to species with at least 100 observations across 10 plots, a total of 56 individual species.

Model Development

A logistic model of the following form was fitted to the data for each species:

\[
\text{UNCR} = \frac{1}{1 + e^{-\alpha x\beta}},
\]

where \(x\beta\) is a linear combination of the predictor variables and \(e\) is the exponential function mathematical constant. Monleon et al. (2004) and Toney and Reeves (2009) found that CCR and the natural logarithm of diameter were the best predictors of UNCR. Given their results, CCR and ln(diameter) were expected to perform well for trees in the South as well. However, because the trees in their studies were mostly conifers and a large proportion of the trees in the South are deciduous hardwoods, the additional variables initially investigated by Monleon et al. and Toney and Reeves were considered for inclusion also: the natural logarithm of total tree length, the length to diameter ratio (ft/in.), and stand-level basal area (ft\(^2\)/ac).

Models were fitted using the SAS procedure NLIN. The averages of the parameter coefficient estimates from Monleon et al. (2004) were used as initial approximations for each parameter. The fitted models were evaluated by visual examination of residual diagnostic plots and with the statistics presented by Toney and Reeves (2009): root mean squared error (RMSE), mean absolute error (MAE), mean error (ME), and model efficiency (EF). RMSE and MAE are assessments of model precision and ME is an assessment of bias. EF is analogous to \(R^2\) in linear regression, and it measures model performance on a relative scale, with 1 indicating a perfect fit, 0 indicating a fit no better than a simple average, and negative values indicating poor fit (Toney and Reeves 2009). These statistics were calculated as follows:

\[
\text{RMSE} = \left(\frac{1}{n} \sum (y_i - \hat{y}_i)^2\right)^{1/2},
\]

\[
\text{MAE} = \frac{1}{n} \sum |y_i - \hat{y}_i|,
\]

\[
\text{ME} = \frac{1}{n} \sum (y_i - \hat{y}_i),
\]

\[
\text{EF} = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2},
\]

where \(n\) is the number of observations for the species being evaluated, \(y_i\) is the observed UNCR for tree \(i\), \(\hat{y}_i\) is the predicted UNCR (UNCR\textsubscript{pred}) for tree \(i\), and \(\bar{y}\) is the average UNCR for the species being evaluated. Monleon et al. (2004) used weighted nonlinear regression with weights equal to the inverse of UNCR \times (1 - UNCR). Toney and Reeves (2009) followed suit, but weighing improved model fits for only 10 of the 35 species they examined. In this study, weighted regression resulted in poorer fits or no appreciable improvement compared with the unweighted models as observed in the residual diagnostic plots and model performance statistics. Therefore, the final models were not weighted.

Initially, 75% of the data were used for fitting the species-specific models, and the remaining 25% were used for model evaluation. Selection of the best set of predictor variables was based on the performance of these two separate data sets; however, to maximize the sample size, final estimation of the model coefficients was done with the full, undivided data set. Furthermore, after examining the initial model fit statistics, it was determined that some species could be combined to the genus level with minimal loss in predictive power. Final models are presented for 24 individual species and 12 genera (Tables 1 and 2). The genus-level models were expanded to include trees from species that did not meet the original sample size criteria.

In addition to comparing the fit statistics for the two data sets, model performance was further evaluated by considering the difference between UNCR and UNCR\textsubscript{red} in terms of the measurement quality objective (MQO) targeted by FIA. For UNCR, two independent field crews are required to be within \(\pm 10\%\) for at least 90% of the trees (Schomaker et al. 2007). For example, if one crew assigns a tree a UNCR of 55%, a second crew is within tolerance if it assigns the same tree a UNCR between 45% and 65%, inclusive. Letting the observed and predicted UNCR values represent field calls from two independent field crews a pseudo-MQO compliance rate (MQO\textsubscript{pseudo}) was calculated as the percentage of UNCR\textsubscript{red} values within \(\pm 10\%\) of the observed UNCR. MQO\textsubscript{pseudo} was then compared with the MQO compliance rates of actual field crews.

Results and Discussion

For most species, the addition of predictor variables beyond CCR provided little improvement in terms of the model performance statistics. Averaged across all 56 of the individual species initially modeled, the RMSE was 0.106 UNCR for the model including all five predictor variables, only a slight improvement over the model with CCR alone (RMSE = 0.111 UNCR). When included with CCR, ln(diameter) and ln(length) provided comparable results; RMSE averaged across all species was 0.110 UNCR and
Table 1. Scientific name, common name, and alphabetic group code of the trees included in the analyses.

<table>
<thead>
<tr>
<th>Scientific name</th>
<th>Common name</th>
<th>Code</th>
<th>Scientific name</th>
<th>Common name</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acer spp.</td>
<td>Maple</td>
<td>ACSR</td>
<td>Nyssa aquatica</td>
<td>Water tupelo</td>
<td>NYAQ</td>
</tr>
<tr>
<td>Betula spp.</td>
<td>Birch</td>
<td>BESP</td>
<td>Nyssa biflora</td>
<td>Swamp tupelo</td>
<td>NYBI</td>
</tr>
<tr>
<td>Carpinus caroliniana</td>
<td>American hornbeam</td>
<td>CARB</td>
<td>Nyssa sylvatica</td>
<td>Black gum</td>
<td>NYSY</td>
</tr>
<tr>
<td>Carya spp.</td>
<td>Hickory</td>
<td>CASP</td>
<td>Oxycoccus arborescens</td>
<td>Sourwood</td>
<td>OXAR</td>
</tr>
<tr>
<td>Carya texana&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Black hickory</td>
<td>CATE</td>
<td>Pinus spp.</td>
<td>Pine</td>
<td>PISP</td>
</tr>
<tr>
<td>Celtis spp.</td>
<td>Hackberry/Sugarberry</td>
<td>CESP</td>
<td>Prosopis glandulosa</td>
<td>Honey mesquite</td>
<td>PRGL</td>
</tr>
<tr>
<td>Cercis floridana</td>
<td>Flowering dogwood</td>
<td>COFL</td>
<td>Prosopis sericata</td>
<td>Black cherry</td>
<td>PRSE</td>
</tr>
<tr>
<td>Diospyros virginiana</td>
<td>Common persimmon</td>
<td>DIVI</td>
<td>Quercus spp.</td>
<td>Oak</td>
<td>QUSP</td>
</tr>
<tr>
<td>Fagus grandifolia</td>
<td>American beech</td>
<td>FAGR</td>
<td>Quercus cocinea&lt;sup&gt;c&lt;/sup&gt;</td>
<td>Scarlet oak</td>
<td>QUOC</td>
</tr>
<tr>
<td>Fraxinus spp.</td>
<td>Ash</td>
<td>FRSP</td>
<td>Quercus fulva&lt;sup&gt;d&lt;/sup&gt;</td>
<td>Southern red oak</td>
<td>QUAH</td>
</tr>
<tr>
<td>Gordonia lasianthus</td>
<td>Loblolly bay</td>
<td>GOLO</td>
<td>Quercus pagoda&lt;sup&gt;e&lt;/sup&gt;</td>
<td>Cherry bark oak</td>
<td>QUPA</td>
</tr>
<tr>
<td>Juglans nigra</td>
<td>Black walnut</td>
<td>JUNI</td>
<td>Quercus phellos</td>
<td>Willow oak</td>
<td>QUPH</td>
</tr>
<tr>
<td>Juniper spp.&lt;sup&gt;a&lt;/sup&gt;</td>
<td>Juniper (forest spp.)</td>
<td>JUSP-F</td>
<td>Robinia pseudacacia</td>
<td>Black locust</td>
<td>ROPS</td>
</tr>
<tr>
<td>Juniperus spp.&lt;sup&gt;a&lt;/sup&gt;,&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Juniper (woodland spp.)</td>
<td>JUSP-W</td>
<td>Salix palustris</td>
<td>Cabbage palmetto</td>
<td>SAPA</td>
</tr>
<tr>
<td>Juniperus ashei&lt;sup&gt;a&lt;/sup&gt;</td>
<td>Ashe juniper</td>
<td>JUAS</td>
<td>Salix nigra</td>
<td>Black willow</td>
<td>SANI</td>
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<tr>
<td>Liquidambar saccataria</td>
<td>Sweetgum</td>
<td>LIST</td>
<td>Sassafras</td>
<td>Sassafras</td>
<td>SAAL</td>
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<tr>
<td>Liriodendron tulipifera</td>
<td>Yellow-poplar</td>
<td>LITU</td>
<td>Sciadopitys</td>
<td>Cypress</td>
<td>TASP</td>
</tr>
<tr>
<td>Magnolia spp.</td>
<td>Magnolia</td>
<td>MASP</td>
<td>Taxodium spp.</td>
<td>Elm</td>
<td>ULSP</td>
</tr>
</tbody>
</table>

* This species was included in its respective genus level model.
<sup>a</sup> Woodyland species.
<sup>b</sup> *J. virginiana* and *J. virginiana var. silicicola.*
<sup>c</sup> *J. virginiana, J. pinanga, and J. ashei.*

0.108 UNCR, respectively. Because the other variables provided only negligible improvement and for consistency with the models developed for the Interior West (Toney and Reeves 2009) and West (Monleón et al. 2004) FIA regions,CCR and ln(diameter) were selected as the final predictor variables except in the case of the woodland junipers, for which replacing ln(diameter) with ln(length)
markedly improved the model fit. As a result, UNCR can be predicted in the following manner:

1. If CCR is >0.9, then set UNCR\textsubscript{pred} = CCR.
2. If CCR is ≤0.9 and the species is Ashe juniper, then
   \[ UNCR\textsubscript{pred} = \frac{1}{1 + e^{-[1.01416 + 3.8796 \times CCR - 0.0852 \times \ln(\text{diameter})]}}. \]
3. If CCR is ≤0.9 and the species is a woodland juniper, but not Ashe juniper, then
   \[ UNCR\textsubscript{pred} = \frac{1}{1 + e^{-[1.01416 + 3.8796 \times CCR - 0.0852 \times \ln(\text{diameter})]}}. \]
4. If CCR is ≤0.9 and the species is not a woodland juniper, then insert the estimated coefficients from Table 3 into
   \[ UNCR\textsubscript{pred} = \frac{1}{1 + e^{-[1.01416 + 3.8796 \times CCR - 0.0852 \times \ln(\text{diameter})]}}. \]

On the basis of RMSE, the final models performed as well as the models fitted to trees in the western United States. RMSE ranged from 0.062 UNCR for water tupelo to 0.176 UNCR for honey mesquite (Table 4), compared with a range of 0.062–0.151 UNCR in the Interior West (Toney and Reeves 2009). MAE ranged from 0.042 UNCR for cabbage palmetto to 0.136 UNCR for honey mesquite (Table 4), compared with a range of 0.049–0.117 UNCR in the Interior West (Toney and Reeves 2009). Across all models, RMSE averaged 0.114 UNCR, and MAE averaged 0.086 UNCR. Bias (ME) averaged 0 overall and deviated from 0 by no more than 0.002 UNCR for any model (Table 4). EF ranged from 0.27 for honey mesquite to 0.90 for cabbage palmetto and averaged 0.54 overall (Table 4). As might typically be recommended for \( R^2 \), Toney and Reeves (2009) advised cautious use of models with EF values below about 0.49. Of the 36 models presented, 14 had EF values less than 0.49 (Table 4).

Overall, MQO\textsubscript{pred} was greater than the target MQO compliance rate for field crews (i.e., 90%) for water tupelo and cabbage palmetto only (Figure 1). Although 90% is the target compliance rate, recent field evaluations have shown that 2002–2004 actual compliance rates were 81.9% for softwoods and 76.7% for hardwoods nationwide, and 75.3% overall in the South (Westfall et al.)
In addition to water tupelo and cabbage palmetto, MQO_pseudo was greater than the overall observed rate in the South for the ash, loblolly bay, yellow-poplar, swamp tupelo, scarlet oak, and cypress models (Figure 1). Across all of the model evaluation statistics, the models performed consistently best for cabbage palmetto, swamp tupelo, common persimmon, loblolly bay, and the pine and cypress genera. Model performance was consistently good for the cherrybark and willow oaks, as well. With RMSE > 0.15, EF < 0.50, and MQO_pseudo < 75%, the models for Ashe juniper, honey mesquite, and black locust are not recommended, and models meeting one or more of the previous criteria should be used cautiously.

The logistic regression model does not limit UNCR to values \( \geq \text{CCR} \). Occurrences of UNCR < CCR occurred for 11 trees (0.03%). Eight of these were cabbage palmettos with UNCR and CCR both equal to 10 or 15%. The model underpredicted UNCR for these trees with values of 8.9 or 9.5% and 14.1 or 14.9%, respectively. For the remaining three trees, UNCR-CCR combinations were 95–88%, 95–90%, and 90–90%, and UNCR was 80.8, 89.9, and 89.9%, respectively. The rarity of UNCR < CCR suggests that such results should not be a major concern.

UNCR was equal to CCR for only 9.8% of the trees. The average difference between UNCR and CCR was 0.14 overall (0.11 for softwoods and 0.16 for hardwoods). At the extremes, the average difference between UNCR and CCR was 0.06 for cabbage palmetto and 0.37 for honey mesquite. Crown form likely plays a part in these differences. For example, cabbage palmetto is unbranched with foliage occurring only at the bole terminal, whereas honey mesquite has a very variable growth form including both single-stemmed trees and multistemmed shrubs (Steinberg 2001). The effect of the difference between UNCR and CCR was evident in the model fit statistics. For example, RMSE was 0.063 for cabbage palmetto and 0.176 for honey mesquite, the second lowest and highest RMSE values, respectively. In general, species with excursive crown form tend to have fewer gaps in the crown into which foliage can be "compacted," thus making it easier to predict UNCR from CCR. Model fits were very good for many of the species with excursive crown forms, e.g., cypress, swamp tupelo, persimmon, and pine.

**Testing the Model in Secondary Applications**

Given that UNCR = CCR for very few trees, using CCR in place of UNCR may have a significant impact on subsequent applications. To investigate, UNCR, UNCR_p, and CCR were used to estimate height to live crown base and crown width for trees in the study. Height to live crown base was calculated by multiplying the LCR by the total tree length. Crown widths were calculated with the equations presented by Bechtold (2003) but were not estimated for all trees included in this study because either the species was not included by Bechtold or because LCR was not significant in the crown width equation. As a result, some species included in each modeled genus were necessarily excluded. In addition, black hickory, hackberry/sugarberry, loblolly bay, cherrybark oak, cabbage palmetto, and black willow were excluded, as were the birch, woodland juniper, and magnolia genera.

The median difference in height to live crown base when using UNCR versus UNCR_p was -1.0 ft (Figure 2), although differences for individual trees ranged as high as 50.3 ft. The median differences between using CCR versus UNCR or UNCR_p were 5.3 and 6.5 ft, respectively (Figure 2), with the difference for individual trees ranging as high as 62.3 ft. Except in the maximum values, there was very little difference between the three LCRs in terms of estimating largest crown widths (Figure 2). Differences were \( \leq 1.3 \) ft in magnitude at the 25th, 50th, and 75th percentiles. This is likely due to the fact that LCR is only a moderate contributor to the crown width prediction models (Bechtold 2003). Results of these comparisons reflect the varied impact of using CCR instead of UNCR in secondary applications. Depending on the application, the difference may be substantial or inconsequential; therefore, practitioners are encouraged to explore the possible differences when using the different LCR measurements.
Figure 2. Box plots showing the distribution of the pairwise differences when using the observed uncompacted crown ratio (UNCR), predicted uncompacted crown ratio (UNCRpr), and observed compacted crown ratio (CCR) for estimating height to live crown base and crown width. The lower and upper ends of each box indicate the 25th and 75th percentiles, respectively, and the line inside each box indicates the median. Whiskers above and below each box indicate the 90th and 10th percentiles.

Conclusion

Given the models presented by Monleon et al. (2004) and Toney and Reeves (2009), models to predict UNCR based on CCR and other easily measured tree attributes are now available for the West, Interior West, and Southern FIA administrative regions. If similar models are generated for the Northern FIA administrative region, investigators should examine the portability of the models from other regions for species that cross regional boundaries. Overall, the Southern models performed well for the pines and many of the species that grow on mesic or hydric sites (e.g., cypress, swamp tupelo, and cherrybark oak); however, model performance was generally poor for the maple, oak, and hickory genera. Therefore, application of the models in the oak-hickory and mixed upland hardwood forest types of the South may be limited, but their use in the pine and bottomland hardwood forest types should be quite reliable. The consequences of using UNCR instead of CCR depend on the application in which LCR is required. Using UNCR instead of CCR in applications where LCR contributes only a small amount of information may not result in substantial differences; however, in other applications (e.g., estimating the wind speed necessary for a surface fire to ignite a crown fire [Monleon et al. 2004]), the use of CCR may have greater consequences. For most species, the models presented here are adequate for converting CCR to UNCR and should be of particular use to users of FIA data.

Literature Cited


